Macroeconomics

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Contents

Question 1

1. Consider a closed economy IS-PC-MR model. There is a one period lag in the effect of real interest rates on output in the IS curve and the Phillips Curve is based on adaptive expectations for inflation. The economy starts in equilibrium. Suppose that in period t (and in advance of real interest rates being set for period t) there is a permanent positive shock to the IS curve.

(i) Explain the adjustment of real interest rates, output and inflation from period t until the economy returns to equilibrium. (30%)

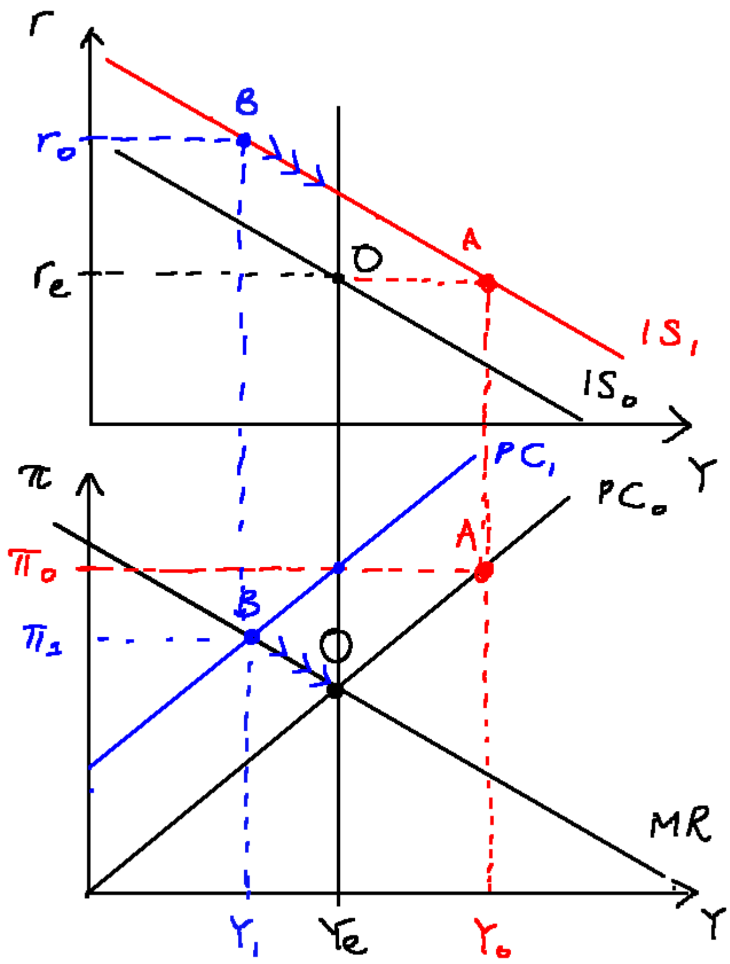

Figure 1: A permanent positive demand shock

Figure 1 illustrates how r, y and π adjust after a permanent positive shock to the IS curve.

$t = 0$. The economy is initially at equilibrium point O, with output on the VPC, and inflation and real rates at their equilibrium rates. The permanent positive demand shock hits, causing the IS to shift from IS_0 to IS_1 (in red). On this new IS curve, the old real interest rate r_e causes an increase in output to Y_0, which in turn leads to higher inflation π_0. The economy thus jumps to point A.

The central bank knows that the IS shift is permanent. It also knows that the AEPC will shift upwards to PC_1 (in blue) so as to intersect the current inflation rate π_0 at the VPC. It therefore wants to locate on point B (blue), the intersection of its MR and the forecasted PC_1 next period. It therefore raises rates to r_0 to decrease output and inflation, but because there is a one period lag in the effect of real interest rates on output nothing happens this period. The economy thus ends period $t = 0$ at point A with high inflation and output.

$t = 1$. The rates now have had time to take effect. As predicted, the PC indeed shifts upwards to PC_1, and the economy jumps to point B on the diagram. At this point, inflation is somewhat elevated but there is a negative output gap. The central bank knows that the PC curve will now fall, and lowers its rates to locate on that new IS curve (not pictured).

$t = 2$ onwards. The PC curve slowly adjusts downwards back to PC_0 as the central bank lowers rates. There is a protracted adjustment back to the equilibrium point O, denoted by the arrows: output slowly recovers and inflation falls back to target.

Now suppose that the same permanent positive IS curve shock occurs in period t (again in advance of interest rates being set for period t), but the central bank initially believes that the shock will last for just one period. Only in period t+1 (and in advance of interest rates being set for period t+1) does the central bank learn that the shock is permanent.

(ii) Explain the adjustment of real interest rates, output and inflation from period t until the economy returns to equilibrium. (30%)

Figure 2 illustrates the case where the CB thinks the shock is temporary.

$t = 0$. The economy is initially at equilibrium point O, with output on the VPC, and inflation and real rates at their equilibrium rates. The permanent positive demand shock hits, causing the IS to shift from IS_0 to IS_1 (in red). On this new IS curve, the old real interest rate r_e causes an increase in output to Y_0, which in turn leads to higher inflation π_0. The economy thus jumps to point A.

The central bank *believes* that the IS shift is temporary. It also knows that the AEPC will shift upwards to PC_1 (in blue) so as to intersect the current inflation rate π_0 at the VPC. Again, it wants to locate on point B (blue), the intersection of its MR and the forecasted PC_1 next period.

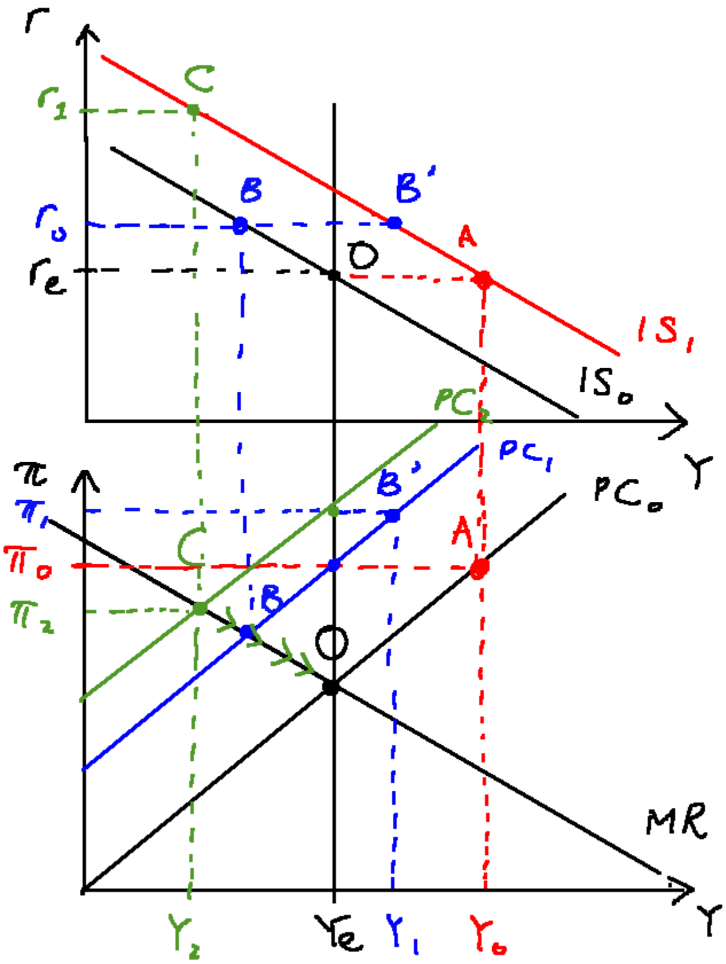

Figure 2: A case of mistaken identity: CB thinks a permanent shock lasts only one period

The key difference lies here. Because the central bank believes that the IS curve will shift back to its original, in order to locate onto point B the central bank sets rates r_0, which is the real rate needed to achieve point B on the old IS curve. We will see how this is a mistake in the next period. Again, because there is a one period lag in the effect of real interest rates on output nothing happens this period. The economy thus ends period $t = 0$ at point A.

$t = 1$. The rates now have had time to take effect. As predicted, the PC indeed shifts upwards to PC_1. But on the new IS curve, the real rate r_0 is too low, actually leads to an *increase* in demand, and thus the economy jumps to point $B^{'}$ on the diagram. The central bank now knows it has made a mistake. It forecasts that the PC will jump again to PC_2 (in green) in the next period, and raises rates to r_1 (on the new IS curve) in order to locate on point C on the MR curve.

$t = 2$ onwards. In $t = 2$, the economy jumps to point C. Again, we have a protracted adjustment back to the equilibrium values.

(iii) Sketch the path followed by inflation over time for the case in which the central bank initially holds incorrect beliefs concerning the duration of the IS curve shock. Explain how the shape of the inflation path is related to the size of the β parameter defining the inflation aversion of the central bank. (40%)

Figure 3 illustrates the path followed by inflation over time. In period $t = 0$, inflation initially starts at its equilibrium value π_e before jumping to π_0 after the shock. The central bank raises rates but not sufficiently, so in period $t = 1$ there is actually a positive output gap which causes inflation to jump again to π_1. Finally in period $t = 2$ the central bank has hiked rates sufficiently such that inflation falls and we have a slow transition to equilibrium after that.

The size of the β parameter will change the shape of the graph, in particular the relative heights of B and C. In the degenerate case, if the bank is infinitely inflation-averse ($\beta \to \infty$) then the MR curve becomes very flat (the slope of MR tends to zero):

$$MR : \pi_t = \pi^T + \frac{y_t - y^T}{\alpha\beta}$$

So even though the bank was mistaken it would still hike rates very very high in $t = 0$ to get zero inflation deviation in the next period. This would mean that point B would be *lower* than A and in fact very close to zero. In general, the more inflation averse the bank, the lower points B and C will be—you may not have a positive output gap in period 1.

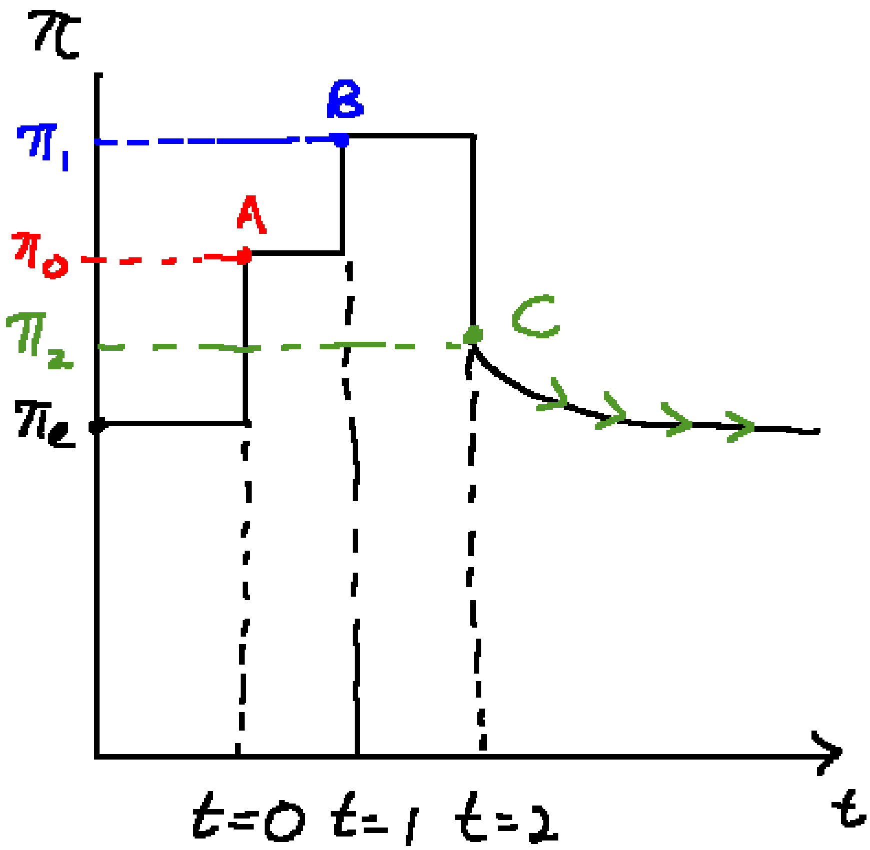

Figure 3: Transition path of mistaken inflation.

Question 2

2. Consider a version of the Solow growth model in which the savings rate and the rate of capital depreciation are positive, and there is no growth in either the level of technology or the size of the workforce.

(i) Explain how the steady-state level of consumption is related to the savings rate and provide economic intuition for this relationship. (40%)

The steady-state level of consumption relates to the savings rate rather ambiguously, and depends on what is known as the "Golden Rule". The closer the savings rate is to the Golden Rule, the higher per capita consumption is.

The Solow model has production in a one-good economy, where goods are produced with both capital and labour according to the Cobb-Douglas production function,

$$Y = AK\alpha N^{1-\alpha}.$$

I now explain how steady-state consumption varies with the savings rate.

Figure 4 illustrates a decrease in the savings rate. A decrease in the savings rate is denoted by a shift in the capital accumulation curve from sY to s_1Y. This causes a drawdown in capital stock, because population growth and capital depreciation remains the same but capital accumulation has decreased. This will cause steady-state per-capita output to fall slowly from y_0 to y_1. However, this does not mean that per capita consumption will decrease. This is because per capita consumption is total output minus what is saved, or $C = (1-s)Y$. In the diagram, steady-state consumption changes from c_0 to c_1, which can be larger or smaller depending on its proximity to the Golden Rule.

What is the economic intuition for this relationship? On one extreme, where no capital is saved, nothing will be produced as K in $Y = K^\alpha L^(1-\alpha)$ will be 0. So there is nothing to consume. But on the other hand, if the society is saving everything that it produces, there will also be nothing to consume. You therefore want to strike a balance between consumption and production.

It turns out that consumption is maximised when the production function and the capital depreciation function have the same slope. The slope of the production function is the MPK, $\alpha A\frac{L^{1-\alpha}}{K^{1-\alpha}}$, and the capital depreciation function has slope δ.

Now suppose that there is a one-off and permanent improvement in the level of technology.

(ii) Explain how the steady-state level of consumption changes in response to the improvement in technology. (30%)

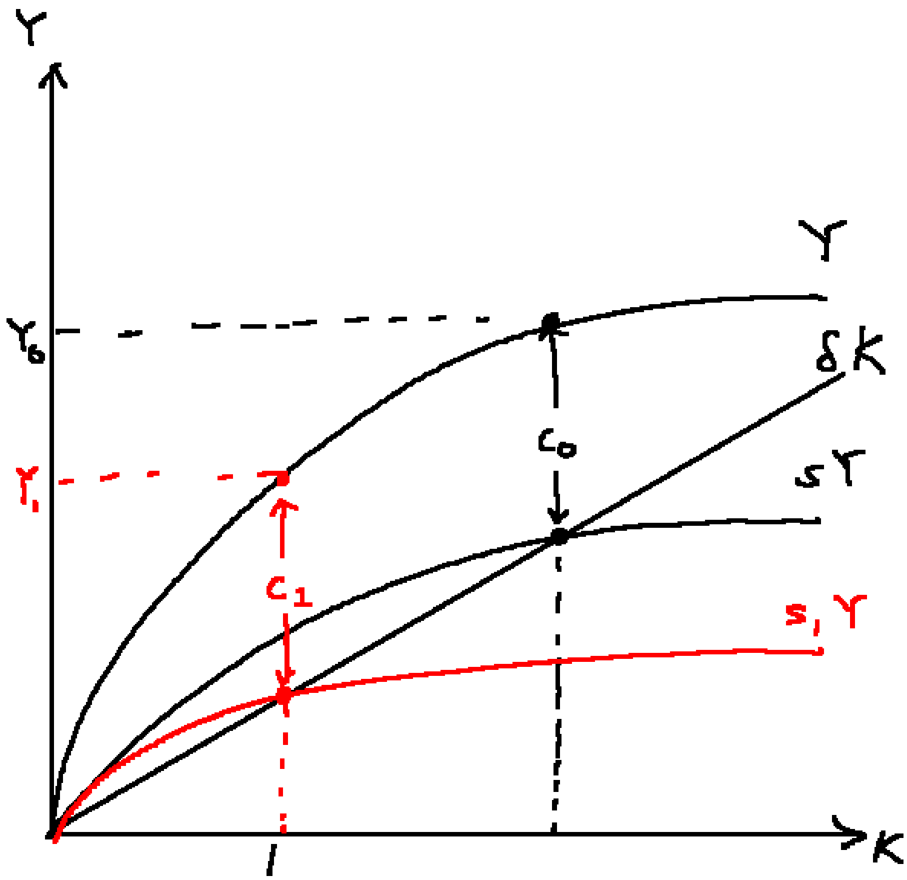

Figure 4: How consumption varies with a change in the savings rate

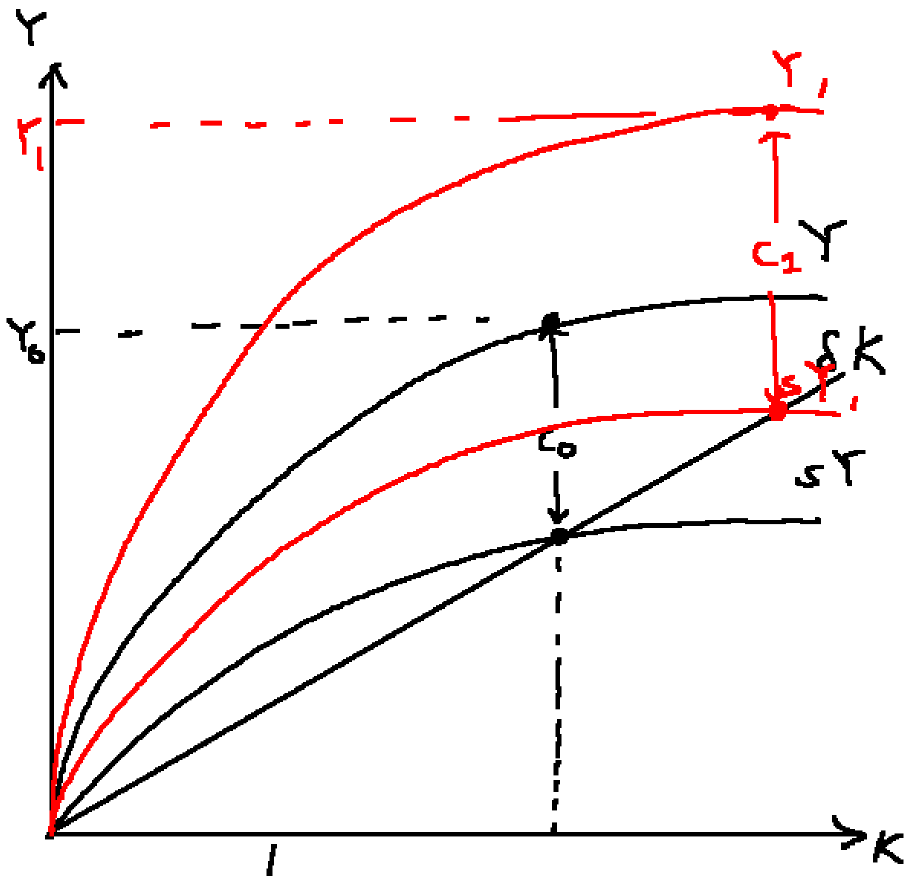

Figure 5:

In Figure 5, a one-off permanent improvement in technology causes output to permanently increase. Because output is governed by the equation

$$Y = AK\alpha N^{1-\alpha},$$

an increase in the technology parameter A causes Y to increase. At this new level of output and steady-state capital, capital accumulates faster than depreciation, and thus there is a slow transition to the new steady-state level of capital and consumption. The new steady-state is the point where the new savings curve sY_1 intersects the capital depreciation line δK.

(iii) Explain how the size of the change to steady-state consumption in part (ii) depends on the rate of capital depreciation and provide economic intuition for your answer. (30%)

The optimal savings rate that maximises steady-state consumption is a value of s such that

$$MPK = \delta,$$

or equivalently,

$$\alpha A \frac{L^{1-\alpha}}{K^{1-\alpha}} = \delta.$$

The change in steady-state consumption thus depends on the value of δ and the savings rate s.

The equation gives us that the higher δ is, the lower should be the steady-state ratio of capital. The economic intuition is intuitive. The more capital depreciates, the less it is worth to accumulate it. The lower δ is (e.g. in the degenerate case, if $\delta = 0$), then you want to accumulate as much capital as you can to produce lots of output.

We can also see from the equation that assuming the savings rate is unchanged, an increase in A means capital-labour ratio $\frac{L}{K}$ rises.

What does this imply?

If the savings rate was previously lower than optimal (which tends to happen when δ is relatively low), then an increase in technology will cause steady-state consumption to increase by a large amount. But if δ is high and the savings rate was already optimal or too high, than an increase in technology causes the capital-labour ratio to increase, which will cause steady-state consumption to increase by a smaller amount.

Question 3

3. Consider the standard intertemporal model of consumption with infinite horizon, rational expectations consumers and perfect capital markets. Consumption is denoted c, the real interest rate is r and financial assets A.

(i) What further assumptions are needed for Hall's random walk result for consumption to hold? Derive the result mathematically and provide economic intuition for the result (include in your answer economic intuition for the roles played by each of the assumptions that you have identified as necessary for the result). (50%)

I derive Hall's random walk in two steps. First, I derive the Euler equation from the intertemporal model of consumption with an infinite horizon. I then derive Hall's random walk from the Euler equation.

To derive Hall's random walk model from the Euler equation, we need the following extra assumptions:

- MU is linear, i.e utility quadratic
- The discount rate β and the interest rate $(1 + r)$ perfectly cancel each other out such that $\beta(1 + r) = 1$.

Deriving the Euler equation

The household's total utility is the expected sum of utilities in the current and all future periods, appropriately discounted with factor β. The total utility can be written as follows:

$$U = E[\sum_{t=0}^{\infty} \beta^t u(c_t)]$$

Constraints of the household

Households own assets or have debt: let us indicate with A_t the household's financial position, which can be positive or negative.

We are given that the interest on debt is the same as the interest rate on savings (the *perfect capital markets* assumption). We thus have that

$$A_{t+1} = (1 + r)A_t,$$

or that the household's assets today is equal its assets tomorrow multiplied by the interest rate.

Additionally, the household has an income in each period of y_t, and chooses consumption of c_t. We therefore get the following equation:

$$A_{t+1} = (1+r)A_t + y_t - c_t$$

This equation means that total assets tomorrow equal assets today (multiplied by the interest rate) plus labour income less consumption. The equation can be consolidated into a single present value constraint, by summing both sides to infinity. That is, on the LHS sum $A_1 + A_2 + A_3 ....$, and on the RHS have $(1+r)A_0 + y_0 - c_0 + (1+r)A_1 + y_1 + c_1 ....$

In order to get the A_t terms to cancel out, we divide each equation by $(1+r)^t$:

$$A_1 = (1+r)A_0 + y_0 - c_0 \tag{1}$$

$$\frac{A_2}{(1+r)} = A_1 + \frac{y_0}{(1+r)} - \frac{c_0}{(1+r)} \tag{2}$$

$$\frac{A_3}{(1+r)^2} = \frac{A_2}{(1+r)} + \frac{y_0}{(1+r)^2} - \frac{c_0}{(1+r)^2} ... \tag{3}$$

The A_t terms cancel out. Rearranging and taking expectations, we obtain:

$$E[\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t}] = E[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}] + (1+r)A_0$$

This is the household's *present value* constraint. The interpretation of this budget constraint is that the expected total future consumption (LHS) must be equal to the total expected income plus whatever initial endowment the household had in the beginning (A_0).

Euler equation

The household maximises expected utility subject to the PV constraint. It can be shown that the first order conditions imply that for any t,

$$MU(c_t) = \beta(1+r)E[MU(c_{t+1})]$$

We have thus derived the *Euler equation*.

From the Euler equation to Hall's random walk model

The Euler equation is an important indifference condition, but it is not yet a consumption function, because we haven't specified a utility function of consumption yet. Now, I show that if we assume that $\beta(1+r) = 1$ and introduce a utility function quadratic in consumption, we get Hall's random walk model: consumption is constant and changes only when there are random shocks.

Suppose the discount factor β and the interest rate $(1+r)$ cancel each other out exactly, such that $\beta(1+r) = 1$. (There is no economic intuition for this assumption, but we get similar results if we don't assume this). Then we can rewrite the Euler equation as

$$MU(c_t) = E[MU(c_{t+1})]$$

That is to say, marginal utility MU is expected to be constant across periods. But consumption may not be, because in general the expected marginal utility from consumption is not equal to the marginal utility from expected consumption. That is,

$$E[MU(c_{t+1})] \neq MU(E[c_{t+1}])$$

We need to assume that utility is quadratic to derive Hall's random walk model. This is needed for a linear marginal utility function. The economic intuition is that households behave as though future income were certain.

Assuming that $u(c) = ac - \frac{b}{2}c^2$ the Euler equation becomes

$$a - bc_t = a - bE[c_{t+1}]$$

Simplifying, we have

$$c_t = E[c_{t+1}],$$

$$c_{t+1} = c_t + \epsilon_t, \quad E[\epsilon_t] = 0$$

This is Hall's Random Walk model of consumption. The economic intuition behind this result is that households consume the same amount in every period, less unexpected and unpredicted shocks. This is a result of consumption smoothing: the household wants to equalise marginal utilities in every period, which given the assumptions of quadratic utility and $(1+r)\beta = 1$ mean equalising consumption in each period.

It immediately follows from this result that $\forall_t c_t = c_0$, which means

$$E[\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t)}] = c_0 \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r} c_0$$

Substituting into the PV budget constraint, we obtain

$$\frac{1+r}{r}c_0 = E[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}] + (1+r)A_0.$$

Rearranging, we obtain

$$c_0 = rA_0 + r[\frac{1}{1+r}E[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}]],$$

or more compactly,

$$c_0 = r[A_0 + H_0] \equiv y_p$$

where

$$H_0 \equiv [\frac{1}{1+r}E[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}]].$$

This result means that households consume in every period a proportion of their permanent wealth, which is the sum of their assets plus their labour income stream. Agents consume a flow of resources that leaves their total wealth unchanged; their total wealth grows at rate r and they consume it at precisely the rate r.

(ii) Show some stuff

We have the present value constraint derived in part (i) as

$$E[\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t}] = E[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}] + (1+r)A_0.$$

Because income can either be y_H or y_L with probability 1/2, the term $E[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}]$ can be rewritten as

By linearity of expectation, we can rewrite as

$$\sum_{t=0}^{\infty} E[\frac{y_t}{(1+r)^t}].$$

And because expected income $E[y_t] = 1/2y_H + 1/2y_L = y^*$, we can simply rewrite

$$\sum_{t=0}^{\infty} \frac{y^*}{(1+r)^t}.$$

Using the formula for the sum of the infinite series we have

$$\sum_{t=0}^{\infty} \frac{y^*}{(1+r)^t} = \frac{1+r}{r} y^*,$$

and substituting into the PV gives us

$$E[\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t}] = \frac{1+r}{r} y^* + (1+r)A_0.$$

which is exactly what we needed to show.

(iii) Suppose households choose c0 after learning whether y0 is yL or yH, but still not knowing what income is going to be thereafter. How much would households consume if they received y0 = yL? What if y0 = y H? In your answers assume that all the further assumptions in part (i) hold, and make use of the random walk result for consumption. (30%)

We have the PV constraint as

$$E[\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t}] = \frac{1+r}{r} y^* + (1+r)A_0.$$

With the random walk result shown earlier, we have

$$\frac{1+r}{r} c_0 = \frac{1+r}{r} y^* + (1+r)A_0.$$

Simplifying,

$$c_0 = y^* + rA_0.$$

This is what households would consume before learning whether y_0 is high or low. Once they know it, as this shock is entirely unexpected, we know that the household will change their consumption by exactly the amount of the shock. That is to say, they will consume

$$c_0 = y^* + rA_0 + (y_H - y_L)$$

if today's income is high, and

$$c_0 = y^* + rA_0 + (y_L - y_H)$$

if today's income is low.

Question 6

6. Growth in the supply of skilled workers has largely outpaced growth in the supply of unskilled workers in many advanced economies since the 1970s, yet the wage premium for skilled workers has substantially increased. With reference to the production technology in the economy, discuss how this outcome might be explained.

Introduction

The law of supply and demand suggests that we should expect the wage premium to fall as the ratio of skilled workers to unskilled workers increases. Yet the wage premium for skilled workers has substantially increased. This seeming paradox can be explained by Acemoglu's model of *directed technical change*. Specifically, this outcome can be explained if high- and low-skilled workers are substitutable. In this essay, I first set up Acemoglu's model with exogenous directed technological change, and show that when high-skilled and low-skilled workers are somewhat substitutable ($\sigma > 1$), the skill premium will increase. I then explain the model that *endogenises* technological change by introducing innovators that produce technology and show that the skill premium will increase if $\sigma > 2$. Finally, I sketch out a way to close the model by discussing the supply-side: as the skill premium increases, the returns to education increase causing the supply of skilled workers to increase further.

Basic setup

In Acemoglu's model of the economy, The labour force is made up of low- (L) and high- (H) skilled workers, who both contribute to producing output Y with their respective technologies A_h and A_l. This model does not include capital. The production function is thus given by:

$$Y = ((A_h H)^\rho + (A_l L)^\rho)^{1/\rho}$$

As this is a competitive labour market, the wages of high- and low- skilled workers must equal their marginal productivities MPH and MPL. Some algebra gives us the following:

$$w_L = \frac{\partial Y}{\partial L} = Ap_l[Ap_l + Ap_h(H/L)p]^{\frac{1-p}{p}}$$
$$w_H = \frac{\partial Y}{\partial H} = Ap_h[Ap_h + Ap_l(H/L)^{-p}]^{\frac{1-p}{p}}$$

The skill premium, ω, is given by the ratio of w_H to w_L. Taking logarithms on both sides, we obtain the skill premium equation:

$$\ln \omega = \frac{\sigma - 1}{\sigma} \ln(\frac{A_h}{A_l}) - \frac{1}{\sigma} \ln(\frac{H}{L})$$

where σ is the elasticity of substitution: $\sigma \equiv \frac{1}{1-\rho}$.

How directed technological change increases the skill premium

How can we explain the empirical finding that the skill premium ω has increased over the decades? The answer is a combination of both elasticity of substitution σ and *skill-biased technical change*. Looking once again at the previous equation, we see that ω can increase if the term $\frac{\sigma-1}{\sigma} \ln(\frac{A_h}{A_l})$ increases. Under what circumstances will this term increase? If we differentiate the skill premium equation, we obtain

$$\frac{\partial \ln w}{\partial A_h / A_l} = \frac{A_l}{A_h} \frac{\sigma - 1}{\sigma}$$

From this equation we can easily see that if $\sigma > 1$ (i.e. $0 < \rho \leq 1$), then

$$\frac{\partial w}{\partial A_h / A_l} > 0$$

i.e., improvements in the skill-complementary technology A_h increase the skill premium. In contrast, when workers are not very substitutable ($\sigma < 1$), an improvement in the productivity of skilled workers *reduces* the skill premium. This case appears paradoxical at first but is in fact quite intuitive. As Acemoglu writes:

> Consider, for example, a Leontieff (fixed proportions) production function. In this case, when A_h increases and skilled workers become more productive, the demand for unskilled workers, who are necessary to produce more output by working with the more productive skilled workers, increases by more than the demand for skilled workers.

Critically, therefore, the skill premium depends on the elasticity of substitution. Figure 6 plots the skill premium equation previously derived. The increase in skilled labour supply H/L causes the skill premium to decrease. This is a movement along the skill premium line from A to B. But if there is skill-biased technology change, this shifts the line upwards. At the same labour supply ratio, the wage premium increases from w^* to w^*. The interpretation here is that the increase in technology, if biased enough, can increase skilled labourers' productivity by more than their increase in supply, causing an increase in skill premium overall.

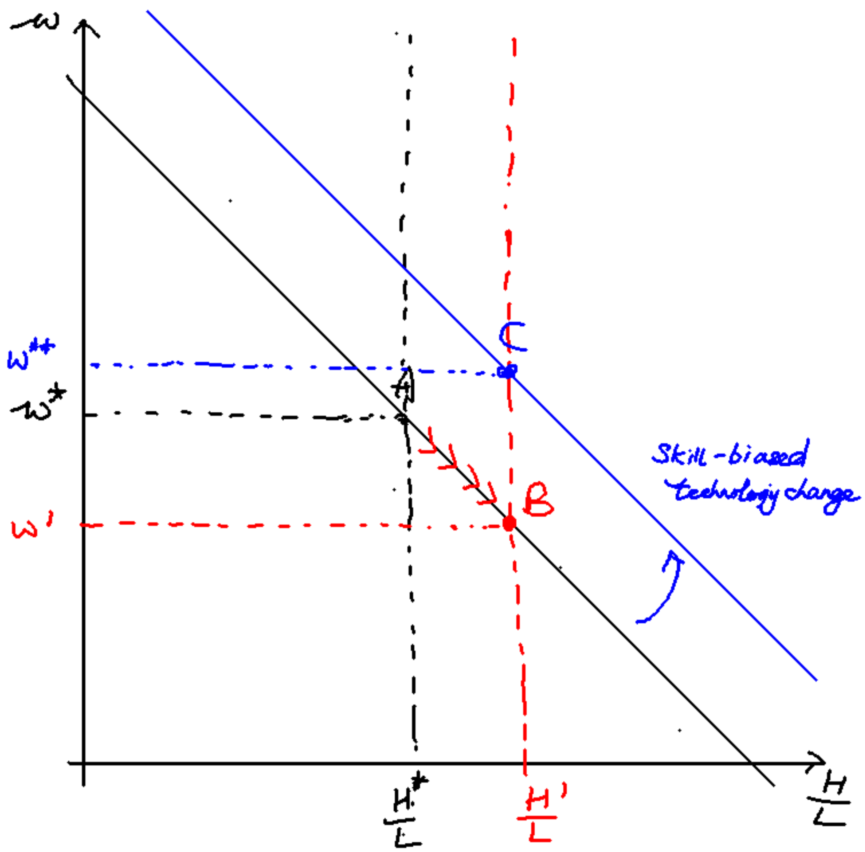

Figure 6: Skill-biased technology change can offset an increase in skilled labour, causing the wage labour to increase

We can thus draw the conclusion that the past 50 years must have been characterised by skill-biased technical change. More explicitly, the relative productivity of skilled workers, $(A_h/A_l)^{\frac{\sigma-1}{\sigma}}$, must have increased.

Endogenising technological development

I've just shown that one can have an increase in the wage premium despite an increase in the ratio of high- to low- skilled workers if the coefficient of substitution $\sigma > 1$ and technical change is directed towards high-skilled workers. But that begs the questions: *why* was technical change directed towards high-skilled workers? In this section I *endogenise* technical change.

We do this by introducing innovators. Innovators can produce innovations, A_H or A_L, that increase the productivity of all high- or low-skill workers. Let the fixed cost of producing an innovation be B and let the marginal cost of producing the new technology be 0. Then the marginal benefit of providing a new technology is simply the marginal increase in productivity (H or L) multiplied by the price of that good; this gives $p_H H$ and $p_L L$ respectively. We can see that innovators will only choose to produce an innovation if they stand to make a profit: in other words only when $B < p_H H$.

Here, there are two factors that affect innovation: price and market size. There is the **price effect**: the larger the price of the good p_H, the more profit innovators stand to make. There is also the **market size effect**: The more high- or low-skilled workers H, the more profit innovators stand to make.

At equilibrium, the marginal benefit of providing both high- and low- skilled technologies must be equal. Otherwise, people would start developing more technologies of the other one to make more profit. This gives us the condition that

$$p_h H = p_l L.$$

By consumer optimisation,

$$\frac{p_H}{p_L} = \frac{MU_H}{MU_L}$$

After all, consider the case where LHS > RHS. Then, it would be a Pareto improvement to substitute 1 unit of H for 1 unit of L.

In this model the production and utility functions are the same. Thus we have that $MU_H == MPH$ and $MU_L = MPL$, and therefore

$$\frac{p_H}{p_L} = \left[\frac{A_H H}{A_L L}\right]^{\rho-1}$$

Substituting in the equilibrium condition, and replacing $\sigma \equiv 1/(1-\rho)$ gives us

$$\frac{A_h}{A_l} = \left(\frac{H}{L}\right)^{\sigma-1}$$

This gives the result that when $\sigma > 1$, more technology will be produced for the higher-skilled worker if H/L increases (i.e. the market size effect dominates). In other words, if $\sigma > 1$, when the proportion of high-skilled workers increases, $H \uparrow$, the amount of technology built for them also increases $\frac{A_H}{A_L} \uparrow$.

Deriving the increased skill premium from endogenous technological change

I've just shown that when $\sigma > 1$, there will be an increase in the ratio of A_h to A_l. Now let us put everything together to explain how both the labour supply and skill premium can increase over time. The skill premium ω can be obtained as follows:

$$\omega \equiv \frac{w_H}{w_L} = \left(\frac{A_H}{A_L}\right)^{\rho} \left(\frac{H}{L}\right)^{-(1-\rho)} \tag{4}$$

Substituting the result we got for $\frac{A_h}{A_l}$ into the equation, as well as the fact that $\sigma \equiv \frac{1}{1-\rho}$, we obtain

$$\omega = \left(\frac{H}{L}\right)^{\sigma-2}$$

We are now able to fully explain the paradox posed in the question! If $\sigma > 2$, then an increase in the relative supply of high-skilled workers increases the skill premium. The wage increase from increased productivity outweighs the wage decrease from increased supply, and both ω and A_H rise.

Figure 7 illustrates such a scenario. The increase in relative supply causes the wage-premium to fall in the short-run. But the *market size effect* encourages innovators to produce more high-skill augmenting technologies, which in turn makes skilled workers more productive and increases their relative wages in the long run.

Extending the model

While Acemoglu does not model explicitly the supply side of workers, if the skill premium is high, then the returns to education will be higher, which will encourage more people to take up education and become high-skilled, which in turn encourages even more skill-biased innovation – a positive feedback loop.

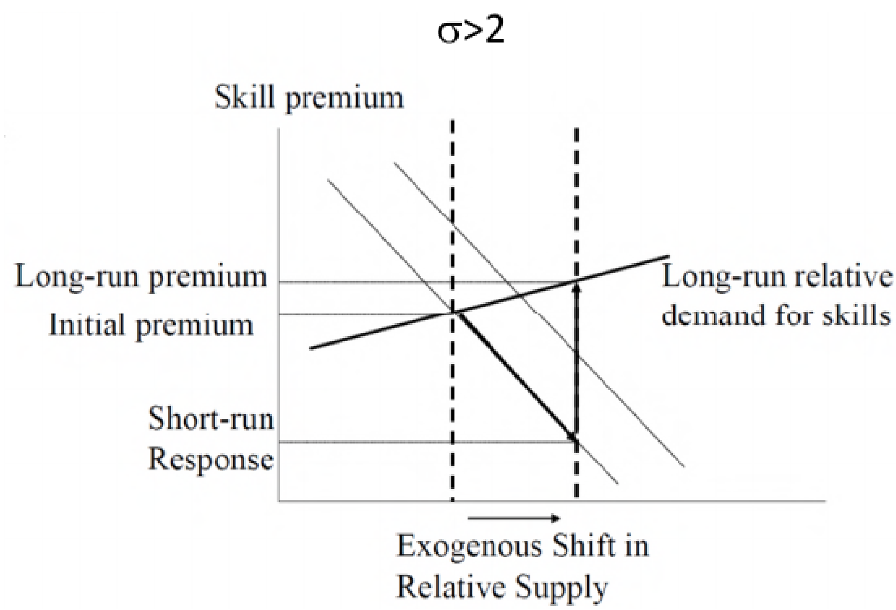

Figure 7: When σ is greater than 2, an increase in the number of high-skilled workers increases both skilled workers productivity and their wage premium

Conclusion

In conclusion, the seeming paradox of increased high-skilled supply and increased skill premium can be reconciled under Acemoglu's model of directed technical change. If $\sigma > 2$ (workers are quite substitutable), then the market size effect outweighs the price effect and the skill premium increases with an increase in supply.

Question 8

8. Explain the role of technology shocks in generating business cycle fluctuations in the Real Business Cycle model. Why is it necessary to assume that such shocks are highly persistent in order to match empirical business cycle statistics?

Introduction

In the RBC model, technology shocks act as a way to change the marginal productivities of capital and labour (MPK and MPL). The change in MPK and MPL in turn affect households' choice of consumption and labour. I first set up the RBC model starting from consumer optimisation, then explain how a positive technology shock affects key macroeconomic variables with and without persistence. Empirically, there is a close and persistent correlation between output and consumption, but we do not get this correlation without persistence as output quickly corrects while consumption remains high due to consumption smoothing. We therefore require persistence in the model to match the empirical data.

Basic model setup

In the RBC model, we assume a simple Cobb-Douglas production function that depends on both labour and capital. Let us first look at the production side. The production function is given by

$$Y_t = A_t F(K_t, L_t)$$

We assume perfect factor markets. Profit maximisation by firms means that wages and interest rates equal the marginal products of labour and capital respectively:

$$r_t = A_t \frac{\partial F(K_t, L_t)}{\partial K_t} \equiv MPK_t$$
$$w_t = A_t \frac{\partial F(K_t, L_t)}{\partial L_t} \equiv MPL_t$$

Importantly, we can see that r and w depend positively on A, but w increases with the K/L ratio while r decreases.

Now let us look at the consumer side. Households optimally allocate time between labour and leisure, and trade-off consumption and saving. All savings are invested and become capital in the next period. Households maximise their intertemporal utility function

$$max \quad E[\sum_{t=0}^{\infty}\beta^t u(C_t, L_t).$$

For brevity I shall not derive the Euler equations here (some of this was already done in Part A). From FOCs for labour and consumption we can derive the following equations:

1. the consumption Euler equation

$$MU(C_t) = \beta E[MU(C_{t+1})(1 - \delta + r_{t+1})].$$

This equation illustrates the trade off between consumption now v. consumption later. The left hand side of the equation is the marginal utility from consuming an additional unit today. But if I consume an additional unit today, I will have one less unit to consume in the future, which will be worth more (or possibly less due to depreciation) due to the real interest rate r_{t+1}. Additionally, because future quantities are uncertain, and I discount future consumption, I take the expectation and multiply it by β.

2. the *intratemporal* labour supply equation

$$w_t MU(C_t) = -MU(L_t)$$

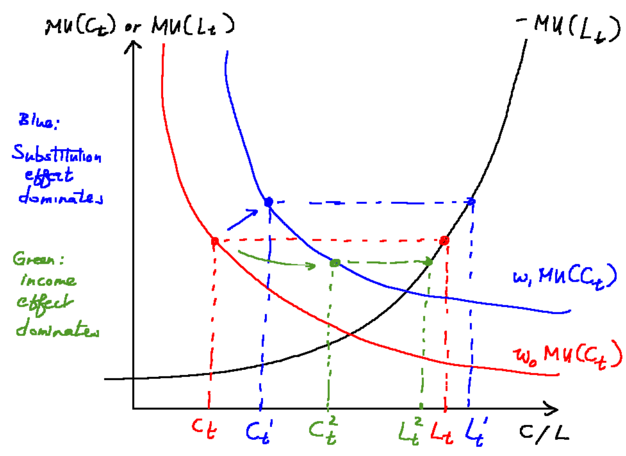

Figure 8: How an increase in wage rate increases the amount of labour supplied

This equation illustrates the tradeoff between work and consumption within a single period. Figure 8 illustrates this intratemporal equation. We plot the decreasing marginal utility of consumption and the increasing marginal cost of labour on the same axes. At a wage rate w_t, the marginal cost from working a little bit more in this period must equal the marginal gain from having more to consume. This is why there is a horizontal dashed line equating the MC and MU.

3. the *intertemporal* labour supply Euler equation obtained by substituting in the intratemporal labour supply equation into the consumption Euler equation:

$$-MU(L_t) = -\beta E[MU(L_{t+1})\frac{w_t}{w_{t+1}}(1-\delta+r_{t+1})].$$

This equation illustrates the tradeoff between working now v. working later. Working more today generates disutility, but it means that my permanent income will increase and I'll be able to consume more (appropriately discounted) in the future.

Model analysis

We have now set up all the moving parts of the RBC model and can now analyse a positive technology shock with and without persistence. We allow TFP, A_t, to be subject to random shocks. Namely we assume $A_t = e^{z_t}$

where $z_t = \rho z_{t-1} + e_t$, $0 \leq \rho \leq 1$.

e is the technology shock and ρ a measure of its persistence.

Let us now analyse a positive technology shock such that $A_0 > 1$. What happens?

Period 0 – no persistence

In period 0, the following occurs:

Firstly, A increases, which increases MPK and MPL. Wages and capital must thus increase in this competitive factor market since they must equal the MPK and MPL.

Households face competing income and substitution effects on consumption. Firstly, permanent income has increased, and thus households will want to increase their overall consumption (income effect). But there is also a substitution effect. Looking at the consumption equation we have

$$MU(C_t) = \beta E[MU(C_{t+1})(1-\delta+r_{t+1})].$$

Here r_{t+1} has increased, which means it is worthwhile to save more (consume less) in this period. We usually assume that the income effect dominates, and so households increase their consumption.

What about labour supply? Households again face competing income and substitution effects. On the one hand, an increase in permanent income means that households want to work less. But on the other hand the ratio of $\frac{w_t}{w_{t+1}}$ in the intertemporal labour supply equation has increased, and so you want to make hay while the sun shines. As Figure 8 shows, labour supply can increase or decrease depending on which effect dominates. Here we assume the substitution effect dominates, given that permanent increases in the real wage in the real world has not led to a decrease in working hours—a temporary wage increase means the income effect is much smaller than a permanent increase in wage.

Thus, consumption and labour supply rises, which causes output to rise. Consumption rises less than output, because it's optimal to consume the increased output over time due to diminishing marginal returns to consumption. Savings also rise because $S = Y - C$ and $\Delta Y > \Delta C$.

Period 1 – no persistence

In period 1, the shock dissipates, and A falls back to normal due to the lack of persistence. However, the economy has accumulated more capital: both by the capital accumulation equation ($K_{t+1} = sY - \delta K_t$: as savings rises, capital accumulated) and the fact that MPK (real interest rate) was higher in the previous period. Due to this fact, MPK goes down (as it is decreasing in K) while MPL remains high (it's increasing in K). Thus we have lower interest rates (lower than steady state r), but elevated wages.

Facing a lowered r, and the fact that they smoothly consume last period's surplus output, households consume more again following the consumption Euler equation.

While wages are very slightly elevated, labour supply actually decreases below equilibrium as the wealth effect outweighs the very small substitution effect. In the intertemporal wage Euler equation

$$-MU(L_t) = -\beta E[MU(L_{t+1})\frac{w_t}{w_{t+1}}(1 - \delta + r_{t+1})],$$

the interest rate this period is lowered which makes working this period less attractive. The very slightly elevated wage ratio $\frac{w_t}{w_{t+1}}$ is too small to matter.

Because people work less in this period, output decreases. The result is that labour supply and output decreases below equilibrium while consumption is elevated for a protracted period of time, as can be seen in the below figure (from slides):

There is no tendency for a period of high output/labour supply to be followed by another period of high output. This is problematic, as empirically i) shocks last a long time and ii) there is a strong correlation between consumption and output.

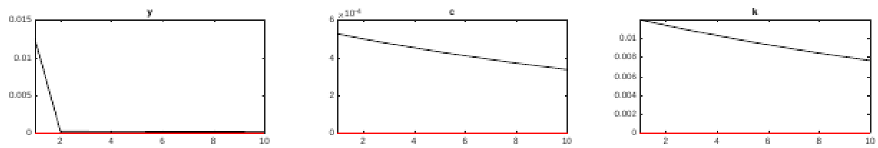

Figure 9: Adjustment of output, consumption and capital with no persistence

A positive productivity shock with persistence

Without persistence, capital accumulation alone does not produce propagation and is insufficient for RBC cyclical behaviour. We thus need persistence where ρ is very close to 1.

This will then give us an extended interval in which productivity is expected to be above normal, and workers respond by increasing their labour supply and accumulating capital. Output and capital will thus track each other closely. This is what happens:

1. A positive productivity shock causes A_0 to increase.
2. This pushes both the MPK r and the MPL w to increase.
3. People thus work more and save more (similar analysis to previous) as long as MPK and MPL remain elevated due to persistence.
4. After a while, the shock starts to dissipate, and the rate of return r decreases below its steady-state level due to the fact that the capital stock is very high and there are diminishing returns to capital.
5. As the rate of return r decreases, and the positive shock starts to dissipate, people start working less: the wealth effect (lots of accumulated capital + low rates) dominates the lower substitution effect (lower wages). Labour supply l falls below its steady-state level.
6. A combination of lower labour supply, elevated consumption, and lower MPK mean that capital starts to decrease until it reaches its previous steady state.

Under this model, we have a protracted adjustment with several periods of output, consumption and labour supply all above trend, which match the empirical correlations in the data much more closely. The figure below (again from the slides) illustrates the persistence, which tracks empirical data much more cleanly.

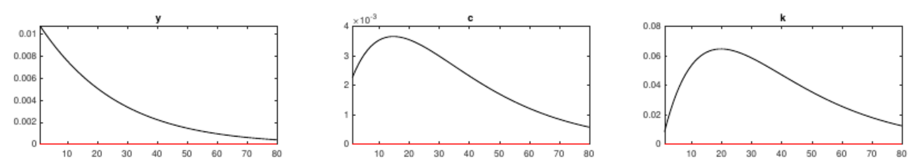

Figure 10: The hump-shaped path of capital and consumption after a positive productivity shock